EFFECT OF TRANSVERSE BLOWING ON STEADY FLOW BETWEEN ROTATING AND STATIONARY DISKS

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A numerical study has been made on the effect of transverse blowing on a steady, incompressible, viscous flow between two infinite coaxial disks, with one of them rotating at an angular speed ω. The fluid is injected through the stationary disk at a constant velocity u. In studying the fluid flow near one infinite rotating disk, assuming self-similar solution for the flow [1], the determination of the velocity components reduces to the solution of the boundary-value problem for the system of ordinary differential equations.

The problem has been well studied in the absence of blowing (see, e.g., [2-8]). Various assumptions on the nature of the flow at high Reynolds number $\mathrm{Re} = \omega d^2/\nu$ (d, distance between the disks; ν, viscosity) were discussed even in the very first papers [2, 3]. According to [2], the flow has two boundary layers attached to the two disks and the fluid in between them rotates as a rigid body.

Arguments in favor of the existence of another type of flow whose characteristic feature is the absence of the rotating fluid outside the boundary layer on the rotating disk have been made in [3]. In what follows, solutions with the above features will be denoted by solution type B and solution type S, respectively.

Later investigations [4-8] showed that there are a number of continuous branches of the solution depending on Re. According to [5], only one of them exists for all $\mathrm{Re} > 0$. For large Re, its solutions are of the type B; it is more convenient to call it the B-branch. The other continuous branch whose solution at large Re are of the type S exists only when $\mathrm{Re} > \mathrm{Re}_1 \cong 217$. Laminar boundary layer velocity profiles observed in experiments at $\mathrm{Re} < 2000$ agree with the numerical results belonging to the B-branch [5-7].

The effect of uniform blowing through the stationary disk on the nature of the flow has been studied in this paper. The solution belonging to the B-branch was chosen for this purpose for a number of Re and it was then continuously extended with respect to the parameter $U = ud/\nu$ (blowing Reynolds number) from $U = 0$ to $U = \mathrm{Re}$.

This problem was already considered in [9, 7] for the range $\mathrm{Re} \leq 100$, where it was shown that with an increase in blowing, the angular and radial components of flow velocity tend to zero in the region outside the boundary layer on the rotating disk while the nature of the flow is continuously varied.

A wider range of Reynolds number $0 < \mathrm{Re} < 700$ is considered in the present paper. It appears that the qualitative behavior of the solution with increased blowing appreciably depends on the value of Re: when $\mathrm{Re} > \mathrm{Re}_* \cong 170$ the continuity in the variation of the flow characteristics with increase in U is broken. Computations for $\mathrm{Re} > \mathrm{Re}_*$ show that for a certain interval (U_1, U_2) of U, there are three solutions belonging to one continuous branch. A branching takes place at the boundaries of this interval: A pair of solutions appear at one end and on the other it disappears.* The curves describing the flow as a function of blowing velocity U become S-shaped. According to the well-known conclusions of bifurcation theory, the upper and the lower segments of the curve correspond to stable flow, at least in the class of rotationally symmetric disturbances, and the middle section corresponds to unstable flow.

As U, increasing from zero, passes through the point U_2, the flow structure is suddenly altered. The boundary layer attached to the stationary disk disappears and the flow outside the boundary layer on the rotating disk tends to become purely axial.

*The singularity referred to as collapse in catastrophy theory corresponds to the bifurcation values U_1, U_2. Probably, the most well-known example of such a bifurcation is the collapse of elastic shells. This is a common bifurcation. We observe that the majority of bifurcations encountered in hydrodynamic stability theory (Taylor vortices, Benards convective cells, etc.) is associated with special symmetry and is not the general bifurcation.

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Similar phenomenon is observed when U decreases from U = Re. In the interval (U_1, Re) solutions are characterized by single boundary layer on the rotating disk and an absence of rotation outside it. When passing through the point U_1, the boundary layer on the rotating disk and the rotation in the core of the flow appear suddenly.

1. Formulation of the Problem

Cylindrical coordinates (r, θ, z) are used. Let the disks be located in the planes z = 0 and z = d; the former rotates at a constant angular speed ω, the second is stationary, and the fluid is injected through it at a constant velocity u.

We investigate axisymmetric flows described by Navier–Stokes equations in the layer $0 \le z \le d$, with a velocity field $\mathbf{v} = (v_r, v_\theta, v_z)$

$$v_r = r\omega F(z_1),\ v_\theta = r\omega G(z_1),\ v_z = \nu d^{-1} H(z_1),$$

where $z_1 = z/d$; ν is the kinematic viscosity. Functions F, G, and H satisfy the equation

$$F'' = HF' + \mathrm{Re}(F^2 - G^2) + S,\ G'' = HG' + 2\,\mathrm{Re}\,FG,\ H' = -2\,\mathrm{Re}\,F \tag{1.1}$$

and the boundary conditions

$$\begin{gathered} F(0) = 0,\ G(0) = 1,\ H(0) = 0, \\ F(1) = 0,\ G(1) = 0,\ H(1) = -U, \end{gathered} \tag{1.2}$$

where $\mathrm{Re} = \omega d^2/\nu$ is the Reynolds number; $U = ud/\nu$ is the Reynolds number based on blowing velocity; S is an unknown constant. If the boundary-value problem (1.1) (1.2) is solved, then the pressure p is determined from the equation

$$\frac{p}{\rho} = \frac{\nu^2}{d^2}\left(H' - \frac{1}{2}H^2\right) + \frac{\omega\nu}{2d^2} Sr^2,$$

where ρ is the fluid density.

2. Computational Results

The boundary-value problem (1.1) (1.2) is solved by shooting technique combined with a continuation about the blowing parameter U at fized Re ∈ (0,700). The parameter U was varied from zero, which corresponds to the B-branch solution, to values of the order of Re.

The quantity $W = -G'(0)\,\mathrm{Re}^{-1/2}$ was chosen to represent the nature of the flow. Its dependence on U for Re = 36, 169, 289, and 529 is shown in Fig. 1 (curves 1-4, respectively), where the coordinate axes are $U\mathrm{Re}^{-1/2}$ and W.

At low Re this relation is nearly linear (curve 1 in Fig. 1). However, with an increase in Re its nature is strongly altered. When $\mathrm{Re} > \mathrm{Re}_* \cong 170$, the curves take up the characteristic S-shaped form (curves 3, 4 in Fig. 1). For each $\mathrm{Re} > \mathrm{Re}_*$ there is such an interval (U_1, U_2) of U that for $U \in (U_1, U_2)$ the problem (1.1), (1.2) has three solutions belonging to one continuous branch, and when $U \notin [U_1, U_2]$ there is only one solution. The limits of the interval are the branch points of the solution to the problem (1.1) (1.2): when $U = U_1$, a pair of solutions appear and when $U = U_2$ it disappears (with increase in U).

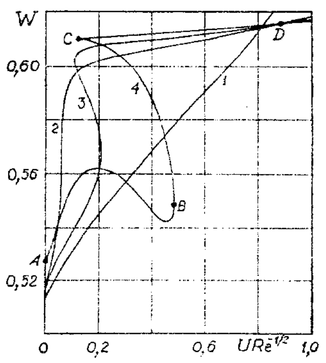

Fig. 1

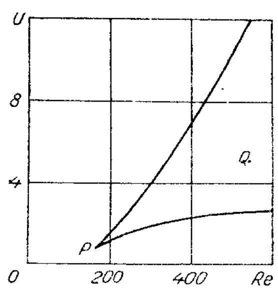

Fig. 2

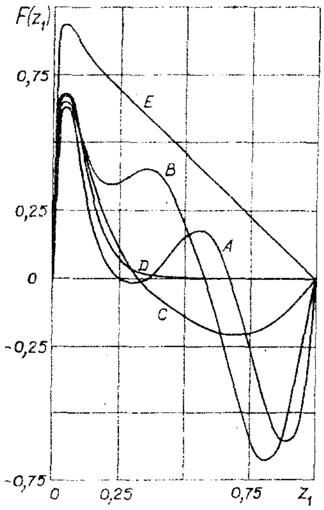

Fig. 3

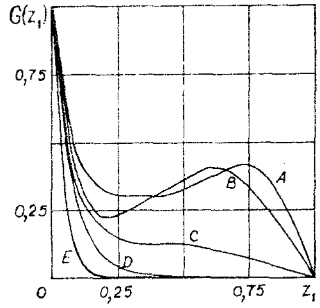

Fig. 4

Points on the plane (Re, U), for which the problem has three solutions (from the family under consideration), form the region Q shown in Fig. 2. At the point of intersection $P(Re_* U_*)$, $U_* = 0.80$, the lines bounding the region make an angle of the order of 3/2.

Let us fix $Re > Re_*$ and investigate how the nature of the flow varies along the continuous branch that depends on the parameter U (for $Re < Re_*$, this was done in [9]). Points A, B, C, and D are indicated in curve 4 in Fig. 1 (Re = 529). The corresponding values of $F(z_1)$ and $G(z_1)$ are shown in Figs. 3 and 4.

The lower parts of the curve 4 (between the points A and B) correspond to the solution of the type B with two boundary layers attached to the disks. In the absence of blowing (point A) the boundary layers are separated by a core in which the fluid rotates like a rigid body. As we move along the curve 4 from the point A to the point B ($0 \leq U \leq U_2$), there is an increase in the thickness of the boundary layer attached to the stationary disk. The point B corresponds to the merging of the boundary layers. On moving from the point B to the point C ($U_1 \leq U \leq U_2$), the boundary layer near the stationary disk disappears.

Solutions of the type S with one boundary layer attached to the rotating disk correspond to the upper part of the curve 4. On moving along the curve 4 from the point C to the point D the flow in the region outside the boundary layer tends to become purely axial. The point D is given by the solution to the Karman problem on the flow near a freely rotating infinite disk.

Further increase in blowing leads to the appearance of radial outflow of fluid from the axis in the entire region outside the boundary layer of the rotating disk. When U is of the order of $Re^{1/2}$, the radial velocity distribution in this region becomes linear. Rotation is observed only in the boundary layer (curves E in Figs. 3 and 4).

Thus, the case of large Reynolds numbers (Re > 170) is characterized by the following features. Firstly, a small amount of blowing ($0 < U < U_2$) leads not to a decrease in radial and tangential components of the velocity near the stationary disk (as in the case $Re \leq 100$, discussed in [9, 7]), but to an increase in the intensity of flow in it. Secondly, with a monotonic variation in the blowing parameter, a sudden alteration in the flow and the hysterisis phenomenon are observed.

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EXACT SOLUTION FOR A HIGH-TEMPERATURE JET

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High-temperature gas (plasma) jets are widely used in modern technology and the jet is often laminar (see, e.g., [1]). The Dorodnitsyn transformation used in the study of nonisothermal jets [2], is useful for plane flows with certain limitations placed on the thermophysical properties of the gas and, besides, it is difficult to convert the Dorodnitsyn variables to physical coordinates. An exact similarity solution within the framework of boundary-layer approximations is given in this paper for the nonisothermal axisymmetric flow in the region where the temperature at the jet axis is appreciably higher than the temperature at infinity.

The problem describing the efflux of a nonisothermal jet from a cylindrical orifice can be written within the framework of boundary-layer approximations in the form

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial w}{\partial r}=\rho\left(v\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right),\quad \frac{1}{r}\frac{\partial}{\partial r}r\rho v+\frac{\partial}{\partial z}\rho w=0,\quad \rho T=1,\ \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial T}{\partial r}=\mathrm{Pr}\rho\left(v\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right); \tag{1}$$

$$v=\frac{\partial w}{\partial r}=\frac{\partial T}{\partial r}=0\quad \text{at}\quad r=0. \tag{2}$$

$$T=\varepsilon,\quad w=0\quad \text{as}\quad r\to\infty. \tag{3}$$

Here r, zRe are cylindrical coordinates (r, z are the inner coordinates in the asymptotic expansion in terms of the small parameter Re^{-1}); $\mathrm{Re}=\sqrt{\rho_M I_{1M}/2\pi}/\mu_M$ is a certain analogous Reynolds number; $v\mathrm{Re}^{-1}$, w are the longitudinal and transverse velocity components; $\mathrm{Pr}=c_{pM}\mu_M/\lambda_M$ is the Prandtl number; ε is the value of the temperature at infinity; the rest are conventional quantities. In order to nondimensionalize, the quantities T_M, ρ_M, c_{pM}, μ_M, and λ_M (dimensional quantities are denoted by the subscript M), and also the values of total impulse I_{1M} and flow enthalpy I_{2M} given by the equations

$$I_{1M}=2\pi\rho_M V_M^2 L_M^2\int_0^\infty \rho w^2 r\,dr,\quad I_{2M}=2\pi c_{pM}\rho_M T_M V_M L_M^2\int_0^\infty \rho w(T-\varepsilon)\,r\,dr$$

are assumed specified. The reference scales for the velocity V_M and the length L_M are given by

$$V_M=c_{pM}T_M I_{1M}/I_{2M},\quad L_M=I_{2M}/(c_{pM}T_M\sqrt{2\pi\rho_M I_{1M}}).$$

In writing Eqs. (1) it was assumed that the specific heat, thermal conductivity, and dynamic viscosity are constants. For the problem (1)-(3) the initial conditions should have been fixed at $z=z_0$ but in the present study only similarity solutions will be considered and hence in order to complete the set of equations for the problem (1)-(3), we formulate conditions for the conservation of momentum and enthalpy

$$\int_0^\infty \rho w^2 r\,dr=1,\quad \int_0^\infty \rho w(T-\varepsilon)\,r\,dr=1. \tag{4}$$

The problem (1)-(4) will be considered as $\varepsilon\to 0$. In the zeroth-order approximation in terms of ε, the problem (1)-(4) is transformed to the system of equations (1), boundary conditions (2), and

$$w=T=0\quad \text{as}\quad r\to\infty; \tag{5}$$

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